Unit III: *Probability-Definition of probability, Classical and empirical probability, Addition and Multiplication rule of probability. Conditional probability, Simple problems.*

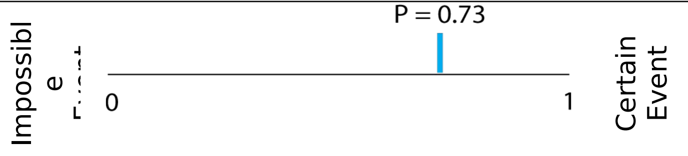
PROBABILITY

PROBABILITY: Probability refers to chance or possibility or likelihood or prospectus that something will occur in future.

Probability = Chance = Possibility = Likelihood = Prospectus

Mathematically, "*probability is a numerical measure of the likelihood that an event will occur*".

1. **Probability of a certain** event is equal to '**1**'.
2. **Probability of an impossible** event is equal to '**0**'.

We represent probability by P. If something is completely certain, P=1.0 If something is absolutely not going to happen, P = 0.0 The real world is always somewhere in between.	

Three Approaches to define Probability

A priori	Probability is estimated by reasoning.	The probability of pulling an ace out of a full deck of cards is 4/52 i.e. 1/13
Empirical	Probability is estimated from data.	Based on recent experience, the probability of a visitor buying something if they visit our site is 0.15 (15%)
Subjective	Probability is based on personal experience and guesswork.	I'm 80% certain that that our store traffic will be down today because it is raining.

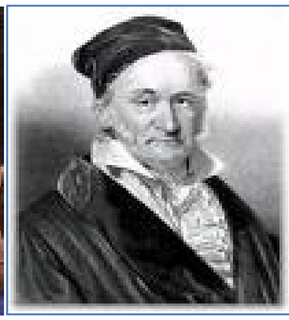
A HISTORICAL NOTE:

1. Galileo (1564-1642) attempted to use quantitative measure to know chance of winning in gambling.
2. First book on the subject was "*Liber de LudoAleave*" i.e. "Book on Games of Chance" published in 1663 written by Jerome Cardano.
3. Theoretical development came during the mid-seventeenth. The chief contributors were
   a. French Mathematician Pascal (1623-1662)
   b. French Mathematician p. Format (1601-1665).
4. During 18th Century

De Moivre (1667-1574)

Thomas Bayes (1702-61)

Pierre Simon deLaplace (1749-1828)



Gauss

Euler

5. In 20^{th} Century

R. A. Fisher

Karl Pearson (1857-1936)

J Neyman

TERMINOLOGY:

An event which is not certain is said to have probable behavior or random behavior. And, the event is known as ***random event***.	1- A card drawn from a deck of card will be an Ace. 2- A household will by a TV on this Festival Season.
A process which results in random event is known as ***random experiment***. Or A process that generates well-defined outcomes but there is an uncertainty that which one will occur is known as a **random experiment** or simply **experiment.**	1- Throwing a fair dice. 2- Tossing a Coin EXPERIMENT — EXPERIMENTAL OUTCOMES 1-Tossing a coin — Head, Tail 2- Rolling a Dice — 1, 2, 3, 4, 5, 6 3-Making a Product — Defect, Not Defect
The set of all possible outcomes of an experiment is called the ***Sample Space.*** *It is denoted by **S.*** Simply, it is the list of all possible experimental outcomes.	Sample Space for flip coin one time=S= {Head, Tail} Sample Space for Rolling a Dice=S= {1, 2, 3, 4, 5, 6} Sample Space for flip coin 2 times=S= {(H, H), (H, T), (T, T), (T, H)}
One of the experimental outcomes is known as **sample point**. A set of some sample points is known as an **event**. A subset of sample space is an **event**.	Experiment: Flip a Coin Sample point=Head Sample point=Tail
*A **random variable*** is one whose exact behavior cannot be predicted in an exact manner, but which may be described in terms of probable behavior. Different random outcomes or random events in a random experiment are the different values of random variables.	1. Consumer durable purchase by a household in this season. Here, different outcomes are possible e.g. household may purchase a TV or a refrigerator or both or none. 2. In rolling a dice outcome is a random variable as any one of the 1, 2, 3, 4, 5, 6 may appear.
The occurrences of an event drawn from occurrence of a series of two or more events are known as ***Joint Event***. A **joint event** is an event that has two or more characteristics jointly.	1- Drawing a red ace from a deck of card is a ***joint event***.
The **complement of an event** A (represented by the symbol A') includes all events that are not part of A. Two events are **complementary** if they cannot occur at the same time and they make up the whole sample space.	1- In rolling a dice event of coming odd number is ***complimentary*** to the event of coming even number.
If occurrence of one event does not affect occurrence of other then events are said to be ***Independent Events***.	In a series of tossing coin two times out comes in first toss are ***independent*** of outcome in 2^{nd} toss and vice versa. So these are independent event.
If occurrence of one event influences the occurrence of other the second event is said to be ***dependent*** on the first event.	In a series of tossing coin two times event of coming two head ***depends*** on its occurrence in first toss. Thus coming two heads depends on its occurrence in first toss.
Two or more events are ***Mutually Exclusive*** if the occurrence of one prevents the occurrence of all other.	In rolling a dice and considering the outcome as an event, all the events are ***mutually exclusive*** because occurrence of one event prevents others.
Two or more events are ***Equally Likely*** if there is no evidence of occurrence of one event over the other.	In tossing an unbiased coin coming Head or Tail are ***equally likely***.
The all possible outcomes in totality are said to be ***Exhaustive Events.***	In a simultaneous throw of two dices there are 36 outcomes which are possible so these 36 events are ***exhaustive events*** in this case

Random Experiment	**Experimental Outcomes (Sample Point)**	**Sample Space**	**Events**

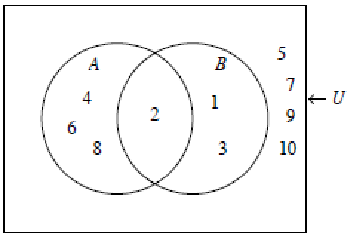
Tossing a Coin	Head, tail	{Head, tail}	Coming Head
Roll a Die	1, 2, 3, 4, 5, 6	{1, 2, 3, 4, 5, 6}	Coming odd number
Tossing Two coins	HH,HT,TH,TT	{HH,HT,TH,TT}	Coming head on one coin
Drawing a card from a deck of card	52-sample points	{,,,,,,,,,,,,,,,,,}	Coming black Jack
Crossing a bridge	Stuck in traffic, Not stuck in traffic	{Stuck in traffic, Not stuck in traffic}	Not stuck in traffic
Today's Weather	?????????	??????????	Stormy rain

SOME BASIC COUNTING PRINCIPLES:

Rule of sum: If an event can be classified into first and second distinct classes with *m* and *n* ways of occurrence respectively. Then, the event has *(m+n)*-ways to occur.	If a bag contain 3-red & 4-blue ball then a ball can be drawn in 3+4=7 ways.
Rule of product/ Multiplication Principle: A procedure can be broken into first and second stages. If the first stage has *m* outcomes and the second stage has *n* outcomes, the total procedure has *(mxn)*-ways. In more general terms, If $c_1, c_2, c_3, \ldots, c_n$, represent the number of choices that can be made for each option then the total number of outcomes will be given by ***Total No. Of outcomes=*** $c_1 \times c_2 \times c_3 \times \ldots \times c_n$	**1.** If a coin is tossed with rolling a die then total number of possible outcome will be 2x6=12. **2.** If three dice are rolled simultaneously then total no of outcome will be 6x6x6=216
Combination Rule: If we take r objects from n distinct objects where $r \le n$ then it can be done in following no. of ways. The total number of ways: ${}^nC_r = \frac{n!}{(n-r)!r!}$	If a bag contains 4 red balls & 7 white balls then 3 white ball can be drawn in ${}^7C_3$-ways. And, the three balls (irrespective of their color) can be drawn in ${}^{11}C_3$-ways.

Set Theoretic Representation

A set is a collection of objects.
We often specify a set by listing its members, or **elements**, in parentheses like this $\{\}$.
For example $A = \{2, 4, 6, 8\}$ means that *A* is the set consisting of numbers 2,4,6,8.
We could also write $A = \{$even numbers less than 9$\}$.
The **union** of *A* and *B* is the set of elements which belong to *A* or to *B* (or both) and can be written as $A \cup B$.
The **intersection** of *A* and *B* is the set of elements which belong to both *A* and *B*, and can be written as $A \cap B$.
The **complement** of *A*, frequently denoted by A^c or $\overline{A}$, is the set of all elements which do not belong to *A*. In making this definition we assume that all elements we are thinking about belong to some larger set *U*, which we call the **universal** set.
The **empty set**, written $\emptyset$ or $\{\}$, means the set with no elements in it.
A set *C* is a **subset** of *A* if **all** the elements in *C* are also in *A*.

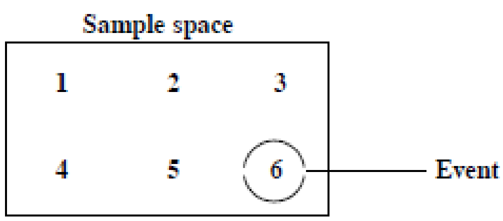


For example, let
$U = \{$all positive numbers $\le 10\}$
$A = \{2, 4, 6, 8\}$
$B = \{1, 2, 3\}$
$C = \{6, 8\}$
Sets *A*, *B* and *U* may be represented in a Venn diagram as on adjacent:

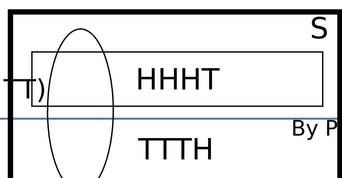
A intersection *B*, $A \cap B$, is shown in the Venn diagram by the overlap of the sets *A* and *B*, $A \cap B = \{2\}$.
The union of the sets *A* and *B*, $A \cup B$, is the set of elements that are in $A = \{2, 4, 6, 8\}$ together with the elements that are in $B = \{1, 2, 3\}$ including each element once only. So, $A \cup B = \{1, 2, 3, 4, 6, 8\}$.
The complement of *A* is the set $\overline{A}$ is contains all the elements in *U* which are not in *A*.
So, $\overline{A} = \{1, 3, 5, 7, 9, 10\}$. *C* is a subset of *A* as every element in $C = \{6, 8\}$ is also in $A = \{2, 4, 6, 8\}$.

In Set theoretic notations, the set of all possible outcomes of the given experiment is called the **sample space**. An **event** is a subset of a sample space.

Consider the e example of rolling a six faced die. The possible outcomes in this experiment are 1,2,3,4,5,6, so the sample space is the set $\{1,2,3,4,5,6\}$. The 'event' of 'getting a 6' is the subset $\{6\}$. We represent this in the following diagram.



Experiment- Tossing two coins.
Sample Points-HH, HT, TH, TT
Sample Space, S= {(H, H), (H, T), (T, H), (T, T)} = (HH, HT, TH, TT)



Event, E_1=Head on first coin. = (HH, HT)
Event E_2= Either Head or tail on both coin= (HH, TT)

BASIC APPROACHES TO DEFINE PROBABILITY

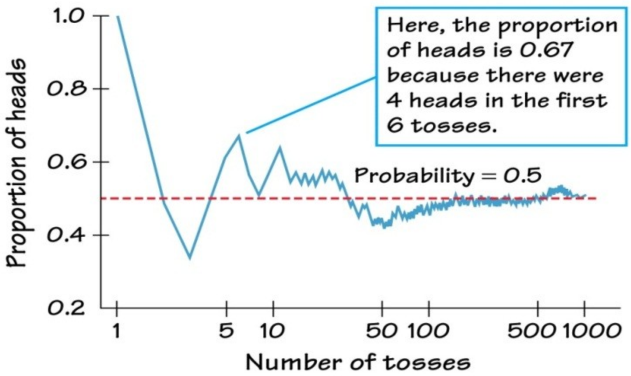
Classical or Priori or Mathematical Definition of Probability: If a trail results in "n" different mutually exclusive, equally likely, exhaustive results and if "f" is the number of favorable cases of happening an Event "E" then the probability of happening Event "E" is given by

$$P(E) = \frac{Favorable\ Number\ of\ Cases}{Exhastive\ Number\ of\ Cases} = \frac{f}{n}$$

Limitation:

1. Events, in practical terms, may not be equally likely.
2. Exhaustive number of cases in a trial may be infinite or uncertain.
3. Depends upon a Priori Analysis.

Empirical or Posterior or Statistical Definition of Probability: If a trail is repeated a number of times under essentially homogeneous & identical conditions, then the limiting value of relative frequency of happening an event, as number of trial becomes infinitely large, is called the probability of happening of the event.



Symbolically, if "n" trials give "f" time favorable outcome for happening of an event E, Then the probability of happening of E is given by

$$P(E) = \lim_{n \to \infty} \frac{f}{n}$$

Limitation:

1. Conditions may not be identical in practice.
2. Infinity concept & its impracticability.
3. Chances of inaccuracy due to infinitely large data.

Modified Mathematical Definition of Probability: The probability P(E) that event E will occur is the ratio of the number of sample points in E to the total number of sample point in S. That is Symbolically,

$$P(E) = \frac{n(E)}{n(S)}$$

Where n(S) = Size of Sample space
n(E)=Size of the event set

Axiomatic Approach to Probability: The axiomatic approach to the probability closely relates the theory of probability with the modern theory of function & set theory. It is developed by Russian mathematician A.N. Kolmogorov in 1933. It includes all the previous theory and also overcomes the drawback of each.

A.N. Kolmogorov

According to this theory, "*The probability of an event A ,given a sample description space S, is a function which assign a non-negative real number to every event A and is denoted by P(A) and is called probability of event A*".

Further P(A) holds following Axioms:

Axiom-I: *For any event A. Probability of A, i.e. P(A)always satisfies*

1. $0 \le P(A) \le 1$
2. $P(A)=1 \Leftrightarrow$ *Event A is certain*
3. $P(A)=0 \Leftrightarrow$ *Event A will never occur*

Axiom-II: *The sum of the probabilities of all of the events in a sample space is 1.i.e. P(S) =1*

Axiom-III: *For any event A, probability of not happening A will be given by* $P(\overline{A}) = 1 - P(A)$

Axiom-IV: *If events A_i (i=1, 2, 3...n) are disjoint events then* $P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$

SOME BASIC ILLUSTRATION

Example 31.1 A die is rolled once. Find the probability of getting a 5.

Solution : There are six possible ways in which a die can fall, out of these only one is favourable to the event.

$\therefore \quad P(5) = \frac{1}{6}$.

Example 31.2 A coin is tossed once. What is the probability of the coin coming up with head ?

Solution : The coin can come up either 'head' (H) or a tail (T). Thus, the total possible outcomes are two and one is favourable to the event.

So, $\quad P(H) = \frac{1}{2}$

Example 31. 3 A die is rolled once. What is the probability of getting a prime number ?

Solution : There are six possible outcomes in a single throw of a die. Out of these; 2, 3 and 5 are the favourable cases.

$\therefore \quad P(\text{Prime Number}) = \frac{3}{6} = \frac{1}{2}$

Example 31.4 A die is rolled once. What is the probability of the number '7' coming up ? What is the probability of a number 'less than 7' coming up ?

Solution : There are six possible outcomes in a single throw of a die and there is no face of the die with mark 7.

$\therefore \quad P(\text{number } 7) = \frac{0}{6} = 0$

[**Note :** That the probability of impossible event is zero]

As every face of a die is marked with a number less than 7,

$\therefore \quad P(\le 7) = \frac{6}{6} = 1$

[**Note :** That the probability of an event that is certain to happen is 1]

Example 31.5 In a simultaneous toss of two coins, find the probability of

(i) getting 2 heads (ii) exactly 1 head

Solution : Here, the possible outcomes are

HH, HT, TH, TT.

i.e., Total number of possible outcomes = 4.

(i) Number of outcomes favourable to the event (2 heads) = 1 (i.e., HH).

$\therefore \quad P(2 \text{ heads}) = \frac{1}{4}$.

(ii) Now the event consisting of exactly one head has two favourable cases, namely HT and TH .

$\therefore \quad P(\text{exactly one head}) = \frac{2}{4} = \frac{1}{2}$.

Example 31.6 In a single throw of two dice, what is the probability that the sum is 9?

Solution : The number of possible outcomes is 6 × 6 = 36. We write them as given below :

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Now, how do we get a total of 9. We have :

$3 + 6 = 9$

$4 + 5 = 9$

$5 + 4 = 9$

$6 + 3 = 9$

In other words, the outcomes (3, 6), (4, 5), (5, 4) and (6, 3) are favourable to the said event, i.e., the number of favourable outcomes is 4.

Hence, $P(\text{a total of } 9) = \frac{4}{36} = \frac{1}{9}$

Example 31.7 From a bag containing 10 red, 4 blue and 6 black balls, a ball is drawn at random. What is the probability of drawing

(i) a red ball ? (ii) a blue ball ? (iii) not a black ball ?

—

Solution : There are 20 balls in all. So, the total number of possible outcomes is 20. (Random drawing of balls ensure equally likely outcomes)

(i) Number of red balls = 10

$\therefore \quad P(\text{a red ball}) = \frac{10}{20} = \frac{1}{2}$

(ii) Number of blue balls = 4

$\therefore \quad P(\text{a blue ball}) = \frac{4}{20} = \frac{1}{5}$

(iii) Number of balls which are not black = 10 + 4

= 14

$\therefore \quad P(\text{not a black ball}) = \frac{14}{20} = \frac{7}{10}$

Example 31.8 A card is drawn at random from a well shuffled deck of 52 cards. If A is the event of getting a queen and B is the event of getting a card bearing a number greater than 4 but less than 10, find P(A) and P (B).

Solution : Well shuffled pack of cards ensures equally likely outcomes.

∴ the total number of possible outcomes is 52.

(i) There are 4 queens in a pack of cards.

$\therefore \quad P(A) = \frac{4}{52} = \frac{1}{13}$

(ii) The cards bearing a number greater than 4 but less than 10 are 5,6, 7,8 and 9.

Each card bearing any of the above number is of 4 suits diamond, spade, club or heart.

Thus, the number of favourable outcomes = 5 × 4 = 20

$\therefore \quad = \frac{20}{52} = \frac{5}{13}$

Example 31.9 What is the chance that a leap year, selected at random, will contain 53 Sundays?

Solution : A leap year consists of 366 days consisting of 52 weeks and 2 extra days. These two extra days can occur in the following possible ways.

(i) Sunday and Monday
(ii) Monday and Tuesday
(iii) Tuesday and Wednesday
(iv) Wednesday and Thursday
(v) Thursday and Friday
(vi) Friday and Saturday
(vii) Saturday and Sunday

Out of the above seven possibilities, two outcomes, e.g., (i) and (vii), are favourable to the event

$\therefore \quad P(53 \text{ Sundays}) = \frac{2}{7}$

Example 31.23 Find the probability of the event getting at least 1 tail, if four coins are tossed once.

Solution : In tossing of 4 coins once, the sample space has 16 samples points.

$\therefore \quad P(\text{at least one tail}) = P(\text{1or 2 or 3 or 4 tails})$

$= 1 - P(0 \text{ tail})$ (By law of complimentation)

$= 1 - P(HHHH)$

The outcome favourable to the event four heads is 1.

$\therefore \quad P(HHHH) = \frac{1}{16}$

Substituting this value in the above equation, we get

$P(\text{at least one tail}) = 1 - \frac{1}{16} = \frac{15}{16}$

In many instances, the probability of an event may be expressed as odds - either odds in favour of an event or odds against an event.

If A is an event :

The odds in favour of $A = \frac{P(A)}{P(\bar{A})}$ or $P(A)$ to $P(\bar{A})$,

where P (A) is the probability of the event A and $P(\bar{A})$ is the probability of the event 'not A'.

Similarly, the odds against A are

$\frac{P(\bar{A})}{P(A)}$ or $P(A)$ to $P(\bar{A})$.

POBABILITY USING COMBINATION RULE

Let us consider the following examples :

Example 31.10 A bag contains 3 red, 6 white and 7 blue balls. What is the probability that two balls drawn are white and blue ?

Solution : Total number of balls = 3 + 6 + 7 = 16

Now, out of 16 balls, 2 can be drawn in ${}^{16}C_2$ ways.

$\therefore$ Exhaustive number of cases = ${}^{16}C_2 = \dfrac{16 \times 15}{2} = 120$

Out of 6 white balls, 1 ball can be drawn in ${}^{6}C_1$ ways and out of 7 blue balls, one can be drawn is ${}^{7}C_1$ ways. Since each of the former case is associated with each of the later case, therefore total number of favourable cases are ${}^{6}C_1 \times {}^{7}C_1 = 6 \times 7 = 42$.

$\therefore$ Required probability = $\dfrac{42}{120} = \dfrac{7}{20}$

Remarks

When two or more balls are drawn from a bag containing several balls, there are two ways in which these balls can be drawn.

(i) **Without replacement :** The ball first drawn is not put back in the bag, when the second ball is drawn. The third ball is also drawn without putting back the balls drawn earlier and so on. Obviously, the case of drawing the balls without replacement is the same as drawing them together.

(ii) **With replacement :** In this case, the ball drawn is put back in the bag before drawing the next ball. Here the number of balls in the bag remains the same, every time a ball is drawn.

In these types of problems, unless stated otherwise, we consider the problem of without replacement.

Example 31.11 Find the probability of getting both red balls, when from a bag containing 5 red and 4 black balls, two balls are drawn,

(i) with replacement.

(ii) without replacement.

Solution : (i) Total number of balls in the bag in both the draws = 5 + 4 = 9

Hence, by fundamental principle of counting, the total number of possible outcomes = 9 × 9 = 81.

Similarly, the number of favourable cases = 5 × 5 = 25.

Hence, probability (both red balls) = $\dfrac{25}{81}$.

(ii) Total number of possible outcomes is equal to the number of ways of selecting 2 balls out of 9 balls = ${}^{9}C_2$.

Number of favourable cases is equal to the number of ways of selecting 2 balls out of 5 red balls = ${}^{5}C_2$.

Hence, P (both red balls) = $\dfrac{{}^{5}C_2}{{}^{9}C_2} = \dfrac{\frac{5 \times 4}{1 \times 2}}{\frac{9 \times 8}{1 \times 2}} = \dfrac{5}{18}$

Example 31.12 Six cards are drawn at random from a pack of 52 cards. What is the probability that 3 will be red and 3 black?

Solution : Six cards can be drawn from the pack of 52 cards in ${}^{52}C_6$ ways.

i.e., Total number of possible outcomes = ${}^{52}C_6$

3 red cards can be drawn in ${}^{26}C_3$ ways and

3 black cards can be drawn in ${}^{26}C_3$ ways.

$\therefore$ Total number of favourable cases = ${}^{26}C_3 \times {}^{26}C_3$

Hence, the required probability = $\dfrac{{}^{26}C_3 \times {}^{26}C_3}{{}^{52}C_6} = \dfrac{13000}{39151}$

Example 31.13 Four persons are chosen at random from a group of 3 men, 2 women and 4 children. Show that the chance that exactly two of them will be children is $\frac{10}{21}$.

Solution : Total number of persons in the group = 3 + 2 + 4 = 9. Four persons are chosen at random. If two of the chosen persons are children, then the remaining two can be chosen from 5 persons (3 men + 2 women).

Number of ways in which 2 children can be selected from 4

children = ${}^{4}C_2 = \dfrac{4 \times 3}{1 \times 2} = 6$

Number of ways in which remaining of the two persons can be selected

from 5 persons = ${}^{5}C_2 = \dfrac{5 \times 4}{1 \times 2} = 10$

Total number of ways in which 4 persons can be selected out of

9 persons = ${}^{9}C_4 = \dfrac{9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4} = 126$

Hence, the required probability = $\dfrac{{}^{4}C_2 \times {}^{5}C_2}{{}^{9}C_4} = \dfrac{6 \times 10}{126} = \dfrac{10}{21}$

Example 31.14 Three cards are drawn from a well-shuffled pack of 52 cards. Find the probability that they are a king, a queen and a jack.

Solution : From a pack of 52 cards, 3 cards can be drawn in ${}^{52}C_3$ ways, all being equally likely.

$\therefore$ Exhaustive number of cases = ${}^{52}C_3$

A pack of cards contains 4 kings, 4 queens and 4 jacks .A king, a queen and a Jack can each be drawn in ${}^{4}C_1$ ways and since each way of drawing a king can be associated with each of the ways of drawing a queen and a jack, the total number of favourable cases = ${}^{4}C_1 \times {}^{4}C_1 \times {}^{4}C_1$

$$\therefore \quad \text{Required probability} = \frac{{}^{4}C_1 \times {}^{4}C_1 \times {}^{4}C_1}{{}^{52}C_3} = \frac{4 \times 4 \times 4}{\frac{52 \times 51 \times 50}{1 \times 2 \times 3}} = \frac{16}{5525}$$

Example 31.15 From 25 tickets, marked with the first 25 numerals, one is drawn at random. Find the probability that it is a multiple of 5.

Solution : Numbers (out of the first 25 numerals) which are multiples of 5 are 5, 10, 15, 20 and 25, i.e., 5 in all. Hence, required favourable cases are = 5.

$\therefore$ Required probability = $\dfrac{5}{25} = \dfrac{1}{5}$

ADDITIVE THEOREM OF PROBABILITIES

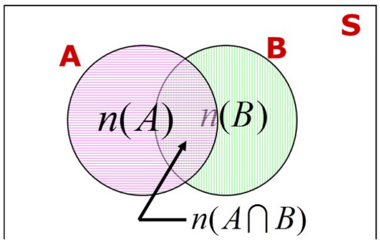
PROBABILITY OF HAPPENING EITHER ONE OR ANOTHER EVENT

Statement: *If A & B are any two events (subset of sample space S) & are not disjoint then*

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)\ or\ P(A\, or\, B)=P(A)+P(B)-P(A\, and\, B)$$

*i.e. Probability of either of two events occurring is the probability for the first event occurring **plus** the probability for the second event occurring **minus** the probability of both event occurring simultaneously.*

Proof: *Symbolically: If A & B are any two events (subset of sample space S)*



We have

$$P(A\cup B)=\frac{n(A\cup B)}{n(S)}=\frac{n(A)+n(B)-n(A\cap B)}{n(S)}$$

$$P(AUB)=\frac{n(A)}{n(S)}+\frac{n(B)}{n(S)}-\frac{n(A\cap B)}{n(S)}$$

$$P(AUB)=P(A)+P(B)-P(A\cap B)$$

SPECIAL CASE: *If A & B are mutually exclusive (i.e.* occurrence of one prevents the occurrence of other*) then*

P(AUB)=P(A) + P(B)

ILLUSTRATION01: A card is drawn from a well-shuffled deck of 52 cards. What is the probability that it is either a spade or a king?

SOLUTION: If a card is drawn at random from a well-shuffled deck of cards, the likelihood of any of the 52 cards being drawn is the same. Obviously, the sample space consists of 52 sample point.

If A and B denote the events of drawing a 'spade card' and a 'king' respectively, then the event A consists of 13 sample points, whereas the event B consists of 4 sample points. Therefore,

P(A)=13/52 P(B)=4/52

The compound event (A∩B) consists of only one sample point, viz.; king of spade. So

P(A∩B)=1/52

Hence, the probability that the card drawn is either a spade or a king is given by

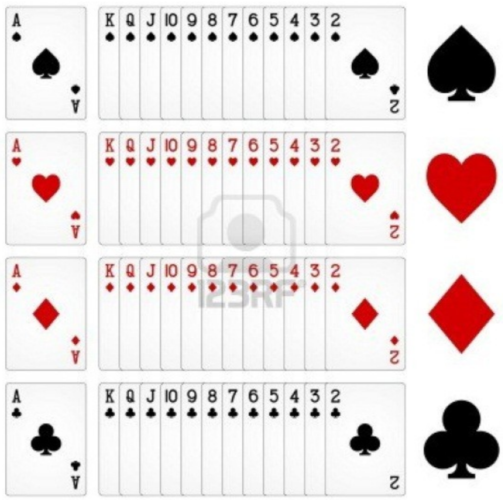
P(AUB) =P(A) + P(B)- P(A∩B)

=13/52+4/52-1/52

=16/52=4/13

ILLUSTRATION02: *Calculate probability of drawing a heart or a jack from an ordinary deck of a card.*

SOLUTION:



P(Heart or Jack) = P(Heart) + P(Jack) – P (Jack with heart)

$¿\frac{13}{52}+\frac{4}{52}-\frac{1}{52}$

$¿\frac{4}{13}$

ILLUSTRATION03: When rolling an honest die, what is the probability that you will roll an even number or a number less than 3?

SOLUTION:

In experiment of rolling a fair die sample space will be

S=(1, 2, 3, 4, 5, 6)⇒n(S)=6

Let us denote

A=outcome is even number=(2, 4, 6))⇒n(A)=3

B=Outcome is a number Less than 3= (1, 2)⇒n(B)=2

As A &B are not disjoint as A∩B= (2) ⇒n (A∩B) =1

P (An even number OR Number less than 3) =P (A∪B) =P (A) +P (B)-P (A∩B) $=\frac{3}{6}+\frac{2}{6}-\frac{1}{6}=\frac{2}{3}$

ILLUSTRATION04: Weather bureau records of a city indicate that 30 per cent of the days are cloudy, 50 per cent are windy and 10 per cent are both cloudy & windy. What is the probability that a day is either cloudy or windy?

SOLUTION:

P(Cloudy or Windy)=P(Cloudy)+P(Windy)-P(Cloudy & Windy)=0.30+0.50-0.10=0.70

There is 70 per cent chance that a day will be either cloudy or windy.

ILLUSTRATION05: A group of students consists of 10 male freshmen, 5 female freshmen, 20 male sophomores, and 18 female sophomores. If one person is randomly selected from the group, find the probability of selecting a freshman or a female.

SOLUTION:

	Male	Female	Total
Freshman	10	5	15
Sophomore	20	18	38
Total	30	23	53

As you see, there are students that can be a female and a freshman at the same time as there are 5 female freshmen students. T

$$P(freshman\ or\ female)=P(freshman)+P(female)-P(female\ freshman)$$
$$=\frac{15}{53}+\frac{23}{53}-\frac{5}{53}=\frac{33}{53}$$

ILLUSTRATION06: In an experiment with throwing 2 fair dice, consider the events

A: The sum of numbers on the faces is 8

B: Doubles are thrown.

What is the probability of getting A or B

SOLUTION: In a throw of two dice, the sample space consists of 6 × 6 = 36 sample points.

The favourable outcomes to the event A (the sum of the numbers on the faces is 8) are

A= { (2, 6) , (3, 5), (4, 4), (5, 3), (6, 2) }

The favourable outcomes to the event B (Double means both dice have the same number) are

B= { (1, 1), (2, 2), (3, 3), (4, 4) ,(5, 5) ,(6, 6) }

Thus A∩B *= { (4, 4) }*

NOW P(A)=5/36 *P(B)=6/36* *P(* A∩B)=1/36

ADDITIVE LAW FOE MUTUALLY EXCLUSIVE/DISJOINT EVENTS

ILLUSTRATION07: A dvd store has 180 comedies, 250 dramas, 230 science fiction movies and 120 mysteries. If you at random select a movie what is the probability that this movie is a comedy or a mystery?

SOLUTION: Since no movie is marked as being a comedy and a mystery there seems to be no overlap in sample spaces. So we use Additive rule of probability for disjoint events and we notice that there is a total of 780 movies.

Example 31.20 In a single throw of two dice, find the probability of a total of 9 or 11.

Solution : Clearly, the events - a total of 9 and a total of 11 are mutually exclusive.

Now $\quad P(\text{a total of } 9) = P[(3,6),(4,5),(5,4),(6,3)] = \frac{4}{36}$

$P(\text{a total of } 11) = P[(5,6),(6,5)] = \frac{2}{36}$

Thus, $\quad P(\text{a total of 9 or 11}) = \frac{4}{36}+\frac{2}{36}$

$= \frac{1}{6}$

Example 31.21 The probabilities that a student will receive an A, B , C or D grade are 0.30, 0.35, 0.20 and 0.15 respectively. What is the probability that a student will receive at least a B grade ?

Solution : The event at least a 'B' grade means that the student gets either a B grade or an A grade.

$\therefore \quad P(\text{at least B grade}) = P(\text{B grade}) + P(\text{A grade})$

$= 0.35 + 0.30$

$= 0.65$

Hence, the probability of at least one green ball is

$P(\text{at least one green ball})$

$= P(\text{one green ball}) + P(\text{two green balls})$

$= \frac{56}{120}+\frac{8}{120}$

$= \frac{64}{120} = \frac{8}{15}$

Let A be the event " at least one green ball is selected".

Let us determine the number of different outcomes in A. These outcomes contain either one green ball or two green balls.

There are ${}^2C_1$ ways to select a green ball from 2 green balls and for this remaining two white balls can be selected in ${}^8C_2$ ways.

Hence, the number of outcomes favourable to one green ball

$= {}^2C_1 \times {}^8C_2$

$= 2 \times 28 = 56$

Similarly, the number of outcomes favourable to two green balls

$= {}^2C_2 \times {}^8C_1 = 1 \times 8 = 8$

$$P(\text{comedy or mystery})=P(\text{comedy})+P(\text{mystery})=\frac{180}{780}+\frac{120}{780}=\frac{300}{780}=\frac{5}{13}$$

MULTIPLICATIVE THEOREM OF PROBABILITIES

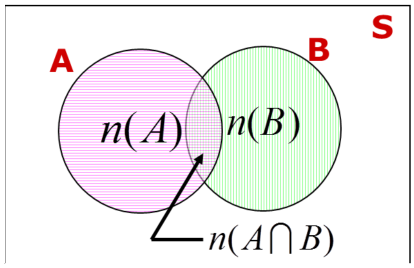
What is the probability of picking two kings from a standard deck of cards? The probability of the first card being a king is= 4/52=1/13
However, the probability of the second card depends on whether or not the first card was a king. If the first card was a king then the probability of the second card being a king is= 3/51=1/17
If the first card was not a king, the probability of the second card being a king is=4/51
THEREFORE, THE SELECTION OF THE FIRST CARD AFFECTS THE PROBABILITY OF THE SECOND CARD.

DEPENDENT EVENTS: When we are looking at probability for two dependent events we need to have notation to express the probability for an event to occur given that another event has already occurred. If A and B are the two events, we can express the probability that B will occur if A has already occurred by using the notation: P(B/A). This notation is generally read as “the probability of B, given A.”

PROBABILITY OF HAPPENING ONE AND ANOTHER EVENT SIMULTANEOUSLY
STATEMENT: *If A & B are any two events (subset of sample space S)*

$$P(A\cap B)=P(A).P(B/A)=P(B).P(A/B) \text{ or equivalently } P(A \text{ and } B)=P(A).P(B/A)=P(B).P(A/B)$$

i.e. Probability of two events occurring simultaneously is the probability for the one event occurring ***Multiply*** *the probability for the other event given first one has already occurred.*
PROOF:
Symbolically: If A & B are any two events (subset of sample space S)



$$\text{We have}$$
$$P(A\cap B)=\frac{n(A\cap B)}{n(S)}$$
$$P(A\cap B)=\frac{n(A)}{n(S)}\frac{n(A\cap B)}{n(A)}, \text{if } A \text{ has already happened}$$
$$P(A\cap B)=P(A).P(B/A)$$
$$\text{Similarly if } B \text{ has already happened we can have}$$
$$P(A\cap B)=P(B).P(A/B)$$

ILLUSTRATION01: A single die is rolled twice. Find the probability of rolling a five the first time and an even number the second time.
SOLUTION: Using Multiplicative theorem of Probability for independent events

$$P(\text{first } a\ 5, \text{ then an even number})=P(\text{first } a\ 5)\cdot P(\text{even number})$$
$$=\frac{1}{6}\cdot\frac{3}{6}=\frac{1}{12} \text{ or } .083$$

ILLUSTRATION02: You are drawing two cards from a regular deck of cards one by one, what is the probability that you draw 2 aces?
SOLUTION: Let A be the event of drawing the first ace and B be the event of drawing the second ace. Note that the two events A & B are not independent events. When we draw the first ace we have 4 aces and 52 cards, so P (A) = 4/52.
After drawing the first ace card, we now have only 3 aces and 51 cards so P (B/A) = 3/51
This gives:
P (2 Aces) = P (First Ace & Second Ace) = P (A∩B)

$$=P(A).P(B/A)=\frac{\frac{4}{52}*3}{51} \quad =1/221$$

ILLUSTRATION03: Six balls in a basket: 2 white, 3 blue and 1 yellow. Two balls are drawn randomly ***one after the other***. What is the probability that the first ball is white, and the second ball is white?
SOLUTION:
Define A = the event the first selected ball is white,
B = the event the second selected ball is white.
What we want: $P(A\cap B)$.
Since P(A) is easy to find (2/6) and P(B|A) is easy to find (1/5), we use the formula:

$$P(A\cap B)=P(A)P(B|A)=(2/6)(1/5)=1/15=0.067$$

Illustration 04: In tossing a coin and rolling a pair of dice what is the probability of a tail and a sum of 4?

Probability of getting Head in tossing a coin

In tossing a coin there are only two outcomes: either a head or a tail appears and favorable outcome to desired event is only one and that is tail

Random Experiment	***Outcome***	
Tossing a coin	Head	Tail

Number of favourable out come=1

Total number of out come=2

P(Getting Tail)=Number of favorable outcome/Total number of possible outcome=1/2

Probability of getting sum of 4 in rolling a pair of dice

In rolling a pair of dice there are total 6x6=36 possible outcome as for each face appeared on first dice there may appear any of the 6 faces on the second dice; and , the and favorable outcome in any of the 3 way as:(1,3),(2,2),(3,1) so that the sum would be 4.

Number of favourable outcome=3

Random Experiment	**Outcome**						
Rolling a pair of dice	Total number of out come=36	Number appeared on other dice					
		1	2	3	4	5	6
Number appered on first dice	1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
	3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
	4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
	6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

P(Sum of 4 on pair of dice)= Number of Favorable Cases/Total Number of Cases=3/36=1/12

Probability of Getting Tail and a Sum of 4

As outcome on the coin and the outcome from pair of dice are independent of each other hence using multiplicative law of probability:

P(Getting Tail & a Sum of 4)=P(Getting Tail) x P(Sum of 4)

P(Getting Tail & a Sum of 4)=(1/2) x (1/12)

P(Getting Tail & a Sum of 4)=(1/24)

P(Getting Tail & a Sum of 4)=(1/24)

ILLUSTRATION05: A store sells 2 different brands of DVD players. Of its DVD player sales, 60% are brand A (less expensive) and 40% are brand B. Each manufacturer offers a 1-yr warranty on parts and labor. It is known that 25% of brand A's DVD players require warranty repair work, whereas 10% for brand B.

(a) What is the probability that a randomly selected purchaser has bought a brand A DVD player that will need repair while under warranty?
(b) (b) Given that the probability that a randomly selected purchaser has a DVD player that will need repair while under warranty is 0.19 (can be obtained from the given conditions). Now if a customer returns to the store with a DVD player that need warranty repair work, what is the probability that it is brand A? Brand B?

SOLUTION:

Define A = event that a randomly selected purchaser bought a brand A DAD player.

B = event that a randomly selected purchaser bought a brand B DAD player.

W = event that purchased DVD play requires repair work.

(a) We are given P(A) = 0.6, P(W|A) = 0.25, P(W|B) = 0.1

And we want: $P(A \cap W)$

$$P(A \cap W) = P(A)P(W|A) = 0.6 \times 0.25 = 0.15$$

(b) We further have P(W)=0.19

And we want: P(A|W) & P(B|W)

$$P(A|W) = \frac{P(A \cap W)}{P(W)} = \frac{0.15}{0.19} = 0.79$$

We can find $P(B \cap W) = P(B)P(W|B) = 0.4 \times 0.1 = 0.04$. So

$$P(B|W) = \frac{P(B \cap W)}{P(W)} = \frac{0.04}{0.19} = 0.21$$

MIXED EXAMPLES:

ILLUSTRATION03: Two cards are dealt from an ordinary deck one by one. What is the probability that at least one is a heart?

SOLUTION: Event "At least one heart" in the experiment of drawing two cards can happen in following mutually disjoint ways:

First card is heart & second card is any other card-(HX)
First card is not a heart but second card is a heart-(XH)
First cart is a heart & second card is also a heart-(HH)

P (At least one H) = P(HX or XH or HH)
= P(HX) + P(XH) + P(HH) As HX, XH & HH are disjoint
= P(H).P(X/H) + P(X).P(H/X) + P(H).P(H/H)
= (13/52).(39/51) + (39/52).(13/51) + (13/52).(12/51)
= (13/52)[39/51+39/51+12/51]
= (1/4)[(39+39+12)/51]
= 15/34

ILLUSTRATION04: A can hit a target 3 times in 5 shots. B 2 times in 5 shots and C 3 times in 4 shots. They fire a volley. What is the probability of hitting 2 shots?

SOLUTION: 2 Hits are possible in following mutually exclusive disjoint ways:

A Hits & B Hits & C does not Hit= (ABC^c)
A Hits & C Hits & B does not Hit= (AB^cC)
B Hits & C Hits & A does not Hit= (A^cBC)

P(2 hit) = $P(ABC^c)+P(AB^cC)+P(A^cBC)$
= $P(A).P(B)\ P(C^c)+$ $P(A).P(B^c).P(C)+$ $P(A^c).P(B).P(C)$ as A,B & C are independent
= (3/5).(2/5)(1/4) + (3/5).(3/5).(3/4) + (2/5).(2/5).(3/4)
= 9/20

ILLUSTRATION05: A problem in Business Statistics is given to four students A, B, C and D. Their respective chances of solving it are 1/2, 1/3, 1/5 and 1/6. What is the probability that the problem will be solved?

SOLUTION:

Event of solving the problem by any student is independent of the event of solving it by any other student.

Let
A=event of solving problem by A & A^c=event that A could not solve the problem
B=event of solving problem by B & B^c=event that B could not solve the problem
C=event of solving problem by C & C^c=event that C could not solve the problem
D=event of solving problem by D & D^c=event that D could not solve the problem

Then P(Problem could not be solved) =P(A^c and B^c and C^c and D^c)
= $P(A^c).\ P(B^c).\ P(C^c).\ P(D^c)$
= (1/2).(2/3).(4/5).(5/6)
= 2/9

P(Problem will be solved) =1 - P(Problem could not be solved)=1 – 2/9=7/9

ILLUSTRATION06: What is the probability of getting more than 10 in a single throw of 2 dice?

SOLUTION:

P(Sum more than 10)= P(Sum is 11 or Sum is 12)
= P(Sum is 11)+ P(Sum is 12) as the event are disjoint
= P[(5,6) or (6,5)] + P[(6,6)]
= P[(5,6)] + P[(6,5)] + P[(6,6)] as the event are disjoint
=(1/36)+ (1/36)+ (1/36)
=1/12

ILLUSTRATION07: Four persons in a group of 20 are graduates. If 4 persons are selected at random from 20, find the probability that

(i) all are graduate
(ii) at least one is a graduate.

SOLUTION:

(i) P(All Graduates)=$1/{}^{20}C_4$=1/9690
(ii) P(No one is graduate)=${}^{16}C_4/{}^{20}C_4$=1820/9690=182/969

P (At Least one graduate)=1- P(No one is graduate)=1-182/969=787/969=0.812

ILLUSTRATION08:

A candidate takes three tests in succession and the probability of passing the first test is 1/2. The probability of passing each succeeding test is 1/2 or 1/4 according as he passes or fails in the preceding one. The candidate is selected if he passes at least two tests. Find the probability that the candidate is selected.

SOLUTION:

Required probability = Probability of passing two tests + Probability of passing all three tests
= P (passing test-I and test-II and fail in test-III) + P (passing test-I, fail in test-II and passing in test-III) + P (fail in test-I, passing in test-II and test-III) + P (passing in all three tests)
=.5*.5*(1-.5)+.5*(1-.5)*.25+(1-.5)*.25*.5+.5*.5*.5
=0.375

ILLUSTRATION08: Five cards are drawn from a pack of 52 cards. Find the probability that

(i) 4 are aces,
(ii) 4 are aces and 1 is a king,
(iii) 3 are kings and 2 are queens,
(iv) a king, queen, jack, 10 and 9 are obtained,
(v) 3 are of any one suit and 2 are of another.

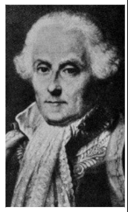
ILLUSTRATION09: The odds that A speaks the truth are 3:2 and the odds that B speaks the truth are 6:3. In what percentage of the cases are they likely to contradict each other on an identical point?

BAYES' THEOREM

THOMAS BAYES (1702-1761)

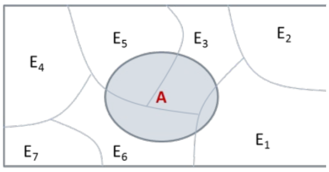
- Born in London, England
- Before Bayes, probability was assumed to have a discrete parameter space.
- "*Bayes invented a new physical model with continuously varying probability of success... He thus gave a geometrical definition of probability as the ratio of two areas.*"
- Only two works published during his life
  1. *Divine Benevolence* (1731)
  2. *Introduction to the Doctrine of Fluxions* (1736)

PIERRE SIMON LAPLACE



- French mathematician
- Responsible for current form of Bayes Theorem
- Bayes found the probability that *x* is between two values given a number of successes and failures
- Laplace found an expression for the probability of a number of future successes and future failures given the number of successes and failures.

"PARTITIONED" SAMPLE SPACES: This means that the sample space is divided into sets where the sets are mutually exclusive, the union of all sets is the whole sample space & there is not an empty set.



Clearly,

$$P(A) = \sum P(E_j)\, P(A|E_j)$$

ILLUSTRATION: In 2010 there will be three candidates for the position of principal -Mr. Chatterji, Mr. Kapoor & Dr. Singh-whose chances of getting the appointment are in the proportion of 4:2:3 respectively. The probability that Mr. Chatterji, if selected, will introduce co-education in the college is 0.3. The probability of Mr. Kapoor& Dr. Singh doing the same are 0.5 & 0.8 respectively. What is the probability that there will be co-education in the college in 2010?

SOLUTION:

Let

A=Introduction of co-education,

E1=Mr. Chatterji is selected as Principal,

E2=Mr. Kapooris selected as Principal,

E3=Dr. Singh is selected as Principal

$$P(A) = \sum P(E_j)\, P(A|E_j)$$

$$P(A) = \frac{4}{9} * \frac{3}{10} + \frac{2}{9} * \frac{5}{10} + \frac{3}{9} * \frac{8}{10} = \frac{23}{45}$$

BAYES' THEOREM: Let events $E_1, \dots, E_k$ form a partition of the space S such that $P(E_j) > 0$ for all i and let A be any event such that $P(A) > 0$. Then for i = 1,..,k:

$$P(E_i | A) = \frac{P(E_i)\, P(A | E_i)}{\sum_j P(E_j) P(A|E_j)}$$

PROOF:

$$P(E_i / A) = \frac{P(E_i \cap A)}{P(A)} = \frac{P(E_i)\, P(A/E_i)}{\sum_j P(E_j) P(A/E_j)}$$

Bayes' Theorem is just a simple rule for computing the conditional probability of events E_i given A from the conditional probability of A given each event E_i and the unconditional probability of each E_i

$P(E_i)$ = Prior distribution for the E_i. It summarizes your beliefs about the probability of event E_i before E_i

$P(A | E_i)$ = The conditional probability of A given E_i. It summarizes the *likelihood* of event A given E_i.

$$P(E_i | A) = \frac{P(E_i)\, P(A | E_i)}{\sum_j P(E_j) P(A|E_j)}$$

$P(E_i | A)$ = The posterior distribution of E_i given A. It represents the probability of event E_i after A has been observed.

$\sum_j P(E_j)\, P(A | E_j)$ = The normalizing constant. This is equal to the sum of the quantities in the numerator for all events E_j. Thus, $P(E_i | A)$ represents the likelihood of event E_i relative to all other elements of the partition of the sample space.

$P(\text{cause}_i)$ = Prior distribution

$P(\text{event} | \text{cause}_i)$ = *likelihood* of event given cause_i.

$$P(\text{cause}_i | \text{event}) = \frac{P(\text{event} \cap \text{cause}_i)}{\sum_j P(\text{event} \cap \text{cause}_j)} = \frac{P(\text{cause}_i).P(\text{event} / \text{cause}_i)}{\sum_j P(\text{cause}_j).P(\text{event} / \text{cause}_j)}$$

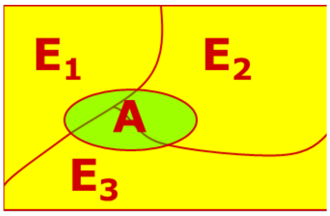
$P(\text{cause}_i | \text{event})$ = The posterior distribution

$\sum_j P(\text{cause}_j)\, P(\text{event} | \text{cause}_j)$ = The normalizing constant.

APPLICATION OF BAYES' THEOREM

ILLUSTRATION01- Bolt machines A, B & C manufacture respectively 25%, 35%, and 40% of the total. Of their output 5,4 & 2% are defective bolts. A bolt is drawn at random from the products & is found to be defective. What is the probability that it was manufactured by Machine A?

SOLUTION:



E_1 denote event of bolt manufactured from machine A
E_2 denote event of bolt manufactured from machine B
E_3 denote event of bolt manufactured from machine C
And Therefore $P(E_1)=.25$ & $P(E_2)=.35$ $P(E_3)=.40$
Further A denote event of bolt being defective.
$P(A/E_1)=.05$, $P(A/E_2)=.04$ & $P(A/E_3)=.02$

Using Bayes theorem,Probability of a Bolt belongs to E_i given that it is defective (i.e. given that it belongs to A)

$$P(E_i \vee A)=\frac{P(E_i)\,P(A \vee E_i)}{P(A)}=\frac{P(E_i)P(A \vee E_i)}{P(E_1)\,P(A \vee E_1)+P(E_2)P(A \vee E_2)+P(E_3)P(A \vee E_3)}$$

(1) Events E_i	(2) Prior Probabilities $P(E_i)$	(3) Conditional Probabilities $P(A \mid E_i)$	(4)=(2)*(3) Joint Probabilities $P(E_i \cap A)$	(5)=(4)/P(A) Posterior Probabilities $P(E_i \mid A)$
E_1	.25	.05	.0125	.0125/.0345=.3623
E_2	.35	.04	.0140	.0140/.0345=.4058
E_3	.40	.02	.0080	.0080/.0345=.2319
	1.00		P(A)=.0345	1.0000

Probability of a defective bolt being from Machine A is 36.23%, from B is 40.58% & from C is 23.19%

ILLUSTRATION02- Records of a company says 20% of its employees are from business schools & of these employees 80% hold a administrative position in the company. Among rest of the employees 30% hold the administrative position. If an administrative staff is selected what is the probability that he had gone to a business school.

SOLUTION:

Let
E_1 denote event of Employee from a business school
E_2 denote event of Employee not from a business school
And Therefore $P(E_1)=.2$ & $P(E_2)=.8$
Further let A denote event of employee being an administrative staff thus $P(A/E_1)=.8$ & $P(A/E_2)=.3$
Using Bayes theorem, Probability of an employee belongs to E_i given that it belongs to administrative position

$$P(E_i \mid A)=\frac{P(E_i)P(A \mid E_i)}{P(E_1)P(A \mid E_1)+P(E_2)P(A \mid E_2)}$$

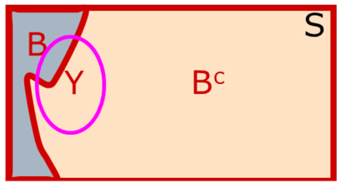
(1) Events E_i	**(2)** Prior Probabilities $P(E_i)$	**(3)** Conditional Probabilities $P(A \mid E_i)$	**(4)=(2)*(3)** Joint Probabilities $P(E_i \cap A)$	**(5)=(4)/P(A)** Posterior Probabilities $P(E_i \mid A)$
E_1	.2	.8	.16	.16/.4=.4
E_2	.8	.3	.24	.24/.4=.6
	1.00		P(A)=.4	**1.0000**

Probability of an administrative staff being from business school is 40%

ILLUSTRATION03- Suppose that 10% of the population in a city is black, and 95% of blacks respond Yes, when asked if they are black in a telephonic survey. Further, 5% of non-blacks respond Yes, when asked if they are black. What is the probability in a survey that someone is black given that they respond that they are black when asked?

SOLUTION:

Suppose that (a)10% of the population is black, so Pr(B) = .10. (b) 95% of blacks respond Yes, when asked if they are black, so Pr(Y_1 | B) = .95. (c) 5% of non-blacks respond Yes, when asked if they are black, so Pr(Y_1 | B^C) = .05



$$\Pr(B/Y_1)=\frac{\Pr(B)\Pr(Y_1/B)}{\Pr(B)\Pr(Y_1|B)+\Pr(B^c)\Pr(Y_1|B^c)}$$

$$\Pr(B/Y_1=1)=\frac{(0.1)(.95)}{(.1)(.95)+(.9)(.05)}=\frac{.095}{.14}=.68$$

Here, we reach the surprising conclusion that even if 95% of black and non-black respondents correctly classify themselves according to race, the **probability that someone is black given that they say they are black is 68%** i.e. less than 70%

ILLUSTRATION04- Consider a manufacturing firm that receives shipment of parts from two suppliers. According to Purchase manager of the firm, firm purchases 65 percent of its parts from supplier 1 and 35 percent from supplier 2. But, quality level differs between suppliers as below:

	PercentageGood Parts	**Percentage Bad Parts**
Supplier 1	98%	2%
Supplier 2	95%	5%

If a bad part is found during manufacturing then what the probability is that it from supplier 1.

SOLUTION:

Let
A_1 denote the event that a part is received from supplier 1;
A_2 is the event the part is received from supplier 2.
G denote that a part is good and *B* denote the event that a part is bad
Thus we have the following probabilities and conditional probabilities:
$P(A_1) = .65$ and $P(A_2) = .35$
$P(G \mid A1) = .98$ and $P(B \mid A1) = .02$
$P(G \mid A2) = .95$ and $P(B \mid A2) = .05$
Using Bayes theorem, Probability of a part being from A_i given that it is bad

$$P(A_i \mid B)=\frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1)+P(A_2)P(B|A_2)}$$

(1) Events ***A_i***	**(2)** Prior Probabilities ***$P(A_i)$***	**(3)** Conditional Probabilities ***$P(B \mid A_i)$***	**(4)=(2)*(3)** Joint Probabilities ***$P(A_i \cap B)$***	**(5)=(4)/P(B)** Posterior Probabilities ***$P(A_i \mid B)$***
A_1	.65	.02	.0130	.0130/.0305=.4262
A_2	.35	.05	.0175	.0175/.0305=.5738
	1.00		P(B)=.0305	**1.0000**

Thus, if a bad part is found during manufacturing then the probability that it from supplier 1 is 42.62%

ILLUSTRATION05-

80% of students pass Chemistry 101. Knowing this, Manuel didn't study for the midterm and ended up failing the midterm. While students who fail the midterm can go on to pass the course, it is a lot harder. Among students who pass the course, only 5% failed the midterm, whereas that percentage goes up to 80% among students who fail the course. Based on this evidence, what is the probability that Manuel will pass Chemistry 101, expressed as a percentage rounded to the nearest integer? (Your answer should be a number between 0 and 100 with no percentage sign.)

Solution:

In standard notation, given that

P(Pass Chemistry 101)=80%=80/100=0.80

P(Fail midterm/Pass Chemistry 101)=5%=0.05

P(Fail midterm/Fail Chemistry 101)=80%=0.80

We can compute

P(Fail Chemistry 101)=1-P(Pass Chemistry 101)=1-0.80=0.20

We can use bays theorem

$$P(E_i \vee A)=\frac{P(E_i)P(A \vee E_i)}{P(E_1)P(A \vee E_1)+P(E_2)P(A \vee E_2)}$$

For the current situation, let A denotes "Fail the midterm", E_1 denote "Pass Chemistry 101" and E_2 denote "Fail Chemistry 101".

Then

$$P(Pass\,Chemistry\,101 | Fail\,Midterm)$$

$$¿\frac{P(Pass\,Chemistry\,101)P(Fail\,Midterm | Pass\,Chemistry\,101)}{P(Pass\,Chemistry\,101)P(Fail\,Midterm | Pass\,Chemistry\,101)+P(Fail\,Chemistry\,101)P(Fail\,Midt}$$

On plugging the values

$$¿\frac{0.80*0.05}{0.80*0.05+0.20*0.80}$$

$$¿0.20$$

$$=20\%$$

Hence,

The probability that Manuel will pass Chemistry given that he has already fail the midterm=20%

Illustration:

A student answers a multiple-choice question with 5 alternatives, of which exactly one is correct. The probability that he knows the correct answer is 1/5. If he does not know the correct answer, he randomly ticks one answer. Given that he has answered the question correctly, find the probability that he did not tick the answer randomly.

Solution:

Let, E_1 = Student knows the answer (note that in this case student will not answer randomly)

E_2 = Student does not know the answer (hence student will Answer randomly)

and A= Student answer correctly.

Thus, we have the following probabilities and conditional probabilities:

$P(E_1)$ = 1/5=.20 and $P(A_2)$ = 1-(1/5)=4/5=.80

$P(A | E_1)$ = 1 (if he knows answer he will surely answer correctly) and $P(A | E_2)$ = 1/5=.20

Using Bayes theorem, Probability of a part being from A_i given that it is bad

$$P(E_i \vee A)=\frac{P(E_i)P(A \vee E_i)}{P(E_1)P(A \vee E_1)+P(E_2)P(A \vee E_2)}$$

(1) Events ***E_i***	**(2)** Prior Probabilities ***$P(E_i)$***	**(3)** Conditional Probabilities ***$P(A \mid E_i)$***	**(4)=(2)*(3)** Joint Probabilities ***$P(E_i \cap A)$***	**(5)=(4)/P(A)** Posterior Probabilities ***$P(E_i \mid A)$***
E_1	.20	1	.20	.20/.36=.5556
E_2	.80	.20	.16	.16/.36=.4444
	1.00		P(A)=.36	**1.0000**

Thus, given that he has answered the question correctly, the probability that he did not tick the answer randomly=.5556

Problems for Practice:

PROBLEM1:15% of the population developed lung cancer and 40% of the country's residents were smokers; 80% of those who developed lung cancer were smokers. What is probability for someone who smoked to develop lung cancer? Set this up using Bayes Theorem and solve.

PROBLEM2: According to a recent Gallup poll, when asked which side of the political debate on abortion do you sympathize with more: the right-to-life or the pro-choice movement, 45% of those polled answered right to life." Of those who declared themselves to be right-to-life, 40% were Democrat, 55% were Republican and 5% were Independent. Of pro-choice, half were Democrat, 45% Republican, 5% were Independent.

- Given a Democrat, what was the probability he or she was pro-choice? Right to life?
- Given a Republican, what was the probability he/she was pro-choice? Right to life?
- Given someone was right to life, what was the probability he/she was Democrat? Republican?

PROBLEM3: Two computers A and B are to be marketed. A salesman who is assigned a job of finding customers for them has 60% and 40% chances of succeeding in case of computers A and B. the computers can be sold independently. Given that he was able to sale at least one computer, what is the probability that the computer A has been sold?

QUESTION BANK

2007	3	Explain the followings: Dependent & Independent Event, Mutually Exclusive Events, Addition & multiplication theory of Probability
2007	5	A problem of Statistics is given to three students A, B & C whose chance of solving it are 1/3, 1/4 and 1/5respectively. Find the probability that the problem will be solved by at least one of them. **(Answer:4/5)**
2007	5	Two unbiased dice are thrown. Find the probability that: 1. Both the dice show the same number. 2. The first dice show six **3.** The total of the number on the dices is 8. **(Answer:1/3,1/6,5/36)**
2007 2008	5 3	A bag contains 3 red, 6 white and 7 blue balls. Two balls are drawn at random. Find the probability that: i. Both balls are white ii. Both balls are blue iii. One ball is red and the other blue iv. One ball is white & other blue.
2007	5	Four cards are drawn from a full pack of cards, Find the probability that: i. There is one card of each suit ii. All the four are spade iii. Two are kings & two are queens
2008	5	A husband & wife appear in an interview for two vacancies in the same post. The probability of husband's selection is 1/7 & that of wife's selection is 1/5. What is probability that (a) both of them will be selected? (b) only one of them will be selected? (c) none of them will be selected? (d) at least one of them will be selected?
2008	5	A problem in business statistics is given to five students A,B,C,D and E. Their chances of solving it are 1/2, 1/3, 1/4, 1/5 and 1/6 . What is the probability that the problem will be solved?
2008	1 0	In a bolt manufacturing factory bolt machines A,B & C manufacture respectively 25%, 35%, and 40% of the total. Of their output 5,4 & 2% are defective bolts. A bolt is drawn at random from the products & is found to be defective. What is the probability that it was manufactured by Machine A, B and C?
2009	3	The probabilities of A, B and C solving a problem are 1/3, 2/7 and 3/8 respectively. If all the three try to solve the problem simultaneously, find the probability that the problem will be solved?
2009	5	A bag contains 4 white, 5 red and 6green balls. Three balls are drawn at random. What is the chance that a white, a red & a green ball is drawn?
2009	5	A bag contains 6 Rupee and 9 dollars coins. Two drawings of 4 coins each are made without replacement. What is the probability that the first draw will give 4 Rupee coins & second 4 dollars coins?
2009	5	Three groups of children contain (i) 3 girls and 1 boy; (ii) 2 girls and 2 boys; (iii) 1 girl and 3 boys. One child is selected at random from each group. Find the probability that the three selected children comprise of 1 girl and 2 boys.
2009	5	The probability that India wins a cricket test match against England is given to be 1/3. If India & and England play 3 test matches what is the probability that: 1. India will lose all the three matches. 2. India will win at least one test match.
2010	3	With the help of an appropriate illustration explain the 'Conditional Probability'.
2010	5	A bag contains 17 tickets marked with number 1 to 17. One ticket is drawn randomly. Find the probability that: i. The number on it is greater than 10. ii. The number on it is multiple of 2 or 5. iii. The number on it is not a multiple of 4.
2010	5	Three cards are drawn in succession and without replacement from an ordinary pack of 52 well shuffled cards. What is the probability that: There will be no queen among the three cards. i. There will be no queen among the cards ii. Only First two cards are aces. iii. First two cards are red & the third card is black.
2010	1 0	There are two urns U_1 & U_2. U_1 Contains 8 red & 6 green balls & U_2 contains 5 red & 7 green balls. Two balls are transferred from U_1 & U_2 and then a ball is drawn from U_2. What is the probability that the ball drawn from U_2 is green ball.
2011	3	A problem in business statistics is given to five students A, B and C. Their chances of solving it are 1/2, 1/3, and 1/4. What is the probability that the problem will be solved?
2011	3	What is the probability of drawing a black card or a king from a pack of ordinary playing cards?
2011	5	A bag contains 30 balls numbered from 1 to 30. One ball is drawn at random. Find the probability that the number of the drawn ball will be a multiple of (a) 5 or 7 and (b) 3 or 7
2011	5	A bag contains 4 white, 2 black, 3 yellow and 3 res balls. What is the probability of getting a white or a red ball at random in the single drawn of one ball?
2011 **2012**	5	A lot contains10 items of which 3 are defective. 3 items are chosen from the lot at random one after another without replacement. Find the probability of all the 3 balls is defective.
2011	5	Suppose it is 9 to 7 against a person who is now 35 years of age living till be is 65 and 3 to 2 against a person now 45 living till be is 75 find the chance that one at least of these person will be alive 30 years hence?
2012	3	Two cards are drawn from a regular deck of cards one by one without the first being replaced, what is the probability drawing 2 aces?
2012	3	A bag contains 6 white, 4 red and 10 black balls. Two balls are drawn at random. Find the probability that they will both be black.
2012	5	Find out the probability of getting a total of either 7 or 11 in a single through of 2 dice.
2012	5	A candidate is selected for interview for three posts. For the first post, there are 3 candidates, for the second there are 4 and for the third there are 2. What are the chances of his getting at least on post?
2012	5	There are two papers in Economics at a certain examination-Paper I and Paper II. The probability that a candidate passes in Paper I is 60% and that he passes in Paper II is 50%. What is the probability that a certain candidate passes only in any one of the two papers?
2013	3	Two dice are thrown once. Find the probability of getting an even number on the first dice or a total of 8.
2013	4	Explain the concept and importance of Probability in business decisions.
2013	6	Discuss the following terms: Random Experiment and sample space, Independent & dependent event.
2013	1 0	A bag contains 5 white and 7 black balls. A ball is drawn out of it and replaced in the bag. Then a ball is drawn again. What is the probability that: i. Both the balls drawn were white ii. Both were Black iii. The first ball was white and second black iv. Both were of the same colour.
2014	1 0	Explain the followings in brief: a. Simple, Composite and compound Interest b. Addition and multiplication theorems c. Sample point and random experiment.
2014	6	A class consists of 100 students, 25 of them are girls and 75 boys; 20 of them are rich and remaining poor; 40 of them are fair complexioned. What is the probability of selecting a fair complexioned rich girl? (Ans: (1/4)*(1/5)*(2/5)=0.02)
2014	4	Find the chance of drawing an ace, a king, a queen and a jack, in this order from an ordinary pack, in four consequtive drawn, the cards drawn are not being replaced
2014	3	The chance of an accident in a factory in a year in the cities A, B and C are 10 in 50, 10 in 120 and 10 in 60 respectively. What is chance that an accident may happen in at least one of them?
2014	3	Two regular sic sided dice are rolled. What is the probability that a six appears on at least one?